

Laplaceova transformacija

$$\mathcal{L}\{y(t)\} = Y(s) \quad Y(s) = \int_0^{\infty} y(t)e^{-st} dt \quad Y(s) = F(s)U(s) \quad y(t) = \frac{1}{2\pi j} \int_{\sigma_c - j\infty}^{\sigma_c + j\infty} Y(s)e^{st} ds$$

Tabela Laplaceovih transformov

Oznake: $A = \sqrt{a^2 + b^2} \quad \omega = \sqrt{b^2 - a^2}$

Lastnosti Laplaceove transformacije

Superpozicija:	$\mathcal{L}\{a_1 y_1(t) + a_2 y_2(t)\} = a_1 Y_1(s) + a_2 Y_2(s)$
Homogenost:	$\mathcal{L}\{ay(t)\} = aY(s)$
Časovni premik:	$\mathcal{L}\{y(t-T)\} = e^{-sT}Y(s)$
Časovno skaliranje:	$\mathcal{L}\{y(at)\} = \frac{1}{ a } Y\left(\frac{s}{a}\right)$
Frekvenčni premik:	$\mathcal{L}\{e^{-at}y(t)\} = Y(s+a)$
Odvajanje:	$\mathcal{L}\left\{\frac{dy}{dt}\right\} = -y(t=0^-) + sY(s)$ $\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s) - s y(t=0^-) - \dot{y}(t=0^-)$ $\mathcal{L}\left\{\frac{d^3y}{dt^3}\right\} = s^3Y(s) - s^2 y(t=0^-) - s \dot{y}(t=0^-) - \ddot{y}(t=0^-)$ $\mathcal{L}\left\{\frac{d^4y}{dt^4}\right\} = s^4Y(s) - s^3 y(t=0^-) - s^2 \dot{y}(t=0^-) - s \ddot{y}(t=0^-) - \dddot{y}(t=0^-)$
Integriranje:	$\mathcal{L}\left\{\int_0^t y(\tau) d\tau\right\} = \frac{1}{s} Y(s)$
Konvolucija v časovnem prostoru ustreza množenju v frekvenčnem prostoru:	$\mathcal{L}\{y_1(t) * y_2(t)\} = Y_1(s)Y_2(s)$
Produkt v čas. prostoru ustreza konvoluciji v frekv. prostoru:	$\mathcal{L}\{y_1(t)y_2(t)\} = \frac{1}{2\pi j} \int_{\sigma_c - j\infty}^{\sigma_c + j\infty} Y_1(\xi)Y_2(s-\xi)d\xi$
Množenje s časom ustreza odvajanju v frekvenčnem prostoru:	$\mathcal{L}\{ty(t)\} = -\frac{d}{ds} Y(s)$

$Y(s)$	$y(t)$
1	$\delta(t)$
$\frac{1}{s}$	1
$\frac{1}{s^2}$	t
$\frac{1}{s^n}; n > 0$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{s-a}$	e^{at}
$\frac{1}{1+sa}$	$\frac{1}{a} e^{-t/a}$
$\frac{1}{s(s-a)}$	$\frac{1}{a}(e^{at} - 1)$
$\frac{1}{s(s+a)}$	$\frac{1}{a}(1 - e^{-at})$
$\frac{1}{s(1+as)}$	$1 - e^{-t/a}$
$\frac{1}{(s-a)^n}; n > 0$	$\frac{t^{n-1}}{(n-1)!} e^{at}$
$\frac{1}{(s-a)(s-b)}$	$\frac{e^{at} - e^{bt}}{a-b}$
$\frac{1}{(1+as)(1+bs)}$	$\frac{e^{-t/a} - e^{-t/b}}{a-b}$
$\frac{1}{s^2 + a^2}$	$\frac{1}{a} \sin(at)$
$\frac{1}{s^2 - a^2}$	$\frac{1}{a} \sinh(at)$
$\frac{s}{s^2 + a^2}$	cos(at)
$\frac{s}{s^2 - a^2}$	cosh(at)
$\frac{1}{s^2 + 2as + b^2}$	$\frac{1}{\omega} e^{-at} \sin(\omega t)$
$\frac{s}{s^2 + 2as + b^2}$	$e^{-at} \left[\cos(\omega t) - \frac{a}{\omega} \sin(\omega t) \right]$
$\frac{1}{s^2(s^2 + 2as + b^2)}$	$\frac{1}{b^2} \left\{ 1 - e^{-at} \left[\cos(\omega t) + \frac{a}{\omega} \sin(\omega t) \right] \right\}$
$\frac{1}{(as+1)^3}$	$\frac{1}{2a^3} t^2 e^{-t/a}$
$\frac{1}{s(s^2 + a^2)}$	$\frac{1}{a^2} [1 - \cos(at)]$
$\frac{as}{(s^2 + a^2)^2}$	$\frac{t}{2} \sinh(at)$

Osnovni linearni členi v regulacijskih sistemih

$$y(t) \xrightarrow{d/dt} x(t) \xrightarrow{\mathcal{L}} Y(s) \xrightarrow{F(s)} X(s)$$

$$F(s) = \frac{X(s)}{Y(s)} = K \frac{\prod_{l=1}^D (1 + sT_l) \cdot \prod_{m=1}^E (1 + 2z_m T_m s + T_m^2 s^2)}{s^{\pm A} \cdot \prod_{n=1}^B (1 + sT_n) \cdot \prod_{r=1}^C (1 + 2z_r T_r s + T_r^2 s^2)}$$

Ime člena	Blok	Prenosna funkcija $F(s) = \frac{X(s)}{Y(s)}$	Zapis v časovnem prostoru (diferencialna enačba)	Opomba	Zgled
Proporcionalni člen		$F(s) = K$	$x(t) = K \cdot y(t)$	K ... ojačenje	uporovni delilnik; idealni ojačevalnik
Integralni člen		$F(s) = \frac{1}{sT_i}$	$x(t) = \frac{1}{T_i} \int_0^t y(\tau) \cdot d\tau$	T_i ... integracijska časovna konstanta	zasuk je integral vrtilne hitrosti (motor)
Člen 1. reda		$F(s) = \frac{K}{1+sT}$	$\frac{dx}{dt} T + x(t) = K \cdot y(t)$	K ... ojačenje T ... časovna konstanta	polnjenje kondenzatorja
Člen 2. reda		$F(s) = \frac{K}{1+2zTs+s^2T^2}$	$\frac{d^2x}{dt^2} T^2 + \frac{dx}{dt} 2zT + x = K \cdot y$	K ... ojačenje T ... časovna konstanta z ... faktor dušenja	RLC nihajni krog
Člen z mrtvim časom		$F(s) = K \cdot e^{-sT_m}$	$x(t) = K \cdot y(t - T_m)$	K ... ojačenje T_m ... mrtvi čas	tekoči trak
Realni diferencialni člen		$F(s) = \frac{sT_d}{1+sT_d}$	$x(t) + \frac{dx}{dt} T_d = \frac{dy}{dt} T_d$	T_d ... časovna konstanta T_d' ... parazitna časovna konstanta	praznjenje kondenzatorja
Idealni diferencialni člen		$F(s) = sT_d$	$x(t) = \frac{dy}{dt} T_d$	T_d ... časovna konstanta ($T_d' = 0$)	prehodna funkcija: Diracov impulz

Člen 1. reda:

$$K = \alpha(\omega=0) \quad T = \frac{1}{\omega_0} \quad \omega_0 = \omega(\varphi = -45^\circ)$$

Člen 2. reda:

$$K = \alpha(\omega=0) \quad T = \frac{1}{\omega_0} \quad \omega_0 = \omega(\varphi = -90^\circ) \quad \omega_r = \omega_0 \sqrt{1-2z^2} \quad \omega_l = \omega_0 \sqrt{1-z^2}$$

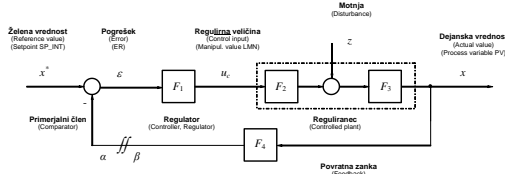
$$\alpha_{\max} = K \cdot Q, \quad Q = \frac{1}{2z\sqrt{1-z^2}}, \quad z = \sqrt{\frac{1 \mp \sqrt{1-Q^2}}{2}}, \quad T = \frac{1}{\omega_0} = \frac{T_d \sqrt{1-z^2}}{2\pi}$$

$$\alpha(\omega_0) = \alpha(\varphi = -90^\circ) = \frac{K}{2z}, \quad x_{\max} = K(1 + e^{-\frac{\pi}{\sqrt{1-z^2}}}), \quad t(x = x_{\max}) = \frac{\pi}{\omega_l}$$

Frekvenčna karakteristika:

$$\alpha(\omega) = |F(j\omega)| = \sqrt{(\text{Re}[F(j\omega)])^2 + (\text{Im}[F(j\omega)])^2} \quad \varphi(\omega) = \arctg\left(\frac{\text{Im}(F(j\omega))}{\text{Re}(F(j\omega))}\right)$$

Prenosne funkcije odprtega in zaključenega regulacijskega kroga



$$F_o = -\frac{\beta}{\alpha} \quad (\text{na mestu prekinitve, } x' = 0, z = 0)$$

$$H = \frac{x}{x'} \quad (z = 0, \text{ sklenjena povratna zanka})$$

$$F_M = \frac{x}{z} \quad (x' = 0, \text{ prekinjena povratna zanka})$$

$$H_M = \frac{x}{z} \quad (x' = 0, \text{ sklenjena povratna zanka})$$

$$F_o = F_1 \cdot F_2 \cdot F_3 \cdot F_4$$

$$H = \frac{F_1 \cdot F_2 \cdot F_3}{1 + F_1 \cdot F_2 \cdot F_3 \cdot F_4} = \frac{F_1 \cdot F_2 \cdot F_3}{1 + F_o}$$

$$F_M = F_3$$

$$H_M = \frac{F_3}{1 + F_1 \cdot F_2 \cdot F_3 \cdot F_4} = \frac{F_M}{1 + F_o}$$

Za direktno povratno zanko (če je $F_4 = 1$) velja tudi:

$$F_o = \frac{x}{x'} \quad (z = 0, \text{ prekinjena povratna zanka})$$

$$H = \frac{x}{x'} \quad (z = 0, \text{ sklenjena povratna zanka})$$

$$F_M = \frac{x}{z} \quad (x' = 0, \text{ prekinjena povratna zanka})$$

$$H_M = \frac{x}{z} \quad (x' = 0, \text{ sklenjena povratna zanka})$$

$$F_o = F_1 \cdot F_2 \cdot F_3$$

$$H = \frac{F_1 \cdot F_2 \cdot F_3}{1 + F_1 \cdot F_2 \cdot F_3} = \frac{F_o}{1 + F_o}$$

$$F_M = F_3$$

$$H_M = \frac{F_3}{1 + F_1 \cdot F_2 \cdot F_3} = \frac{F_M}{1 + F_o}$$

Optimiziranje parametrov regulatorjev

P-regulator: $F_R = K_p$ I-regulator: $F_R = \frac{1}{sT_i}$

PI-regulator: $F_R = K_p \frac{1+sT_{ip}}{sT_{ip}}$ PID-regulator: $F_R = K_p \frac{(1+sT_{ip})(1+sT_{dp})}{sT_{ip}}$

Priloga po Ziegler-Nicholsu za procesne regulacije

	Regulirane	Reg.	Izračun parametrov regulatorja
1	$F_S \approx e^{-\tau_c} \cdot \frac{K_S}{(1+sT')}$	P	$K_p = \frac{T'}{K_S T_m'}$
		PI	$K_p = 0,9 \cdot \frac{T'}{K_S T_m'}, T_{ip} = 3,3 \cdot T_m'$
		PID	$K_p = 1,2 \cdot \frac{T'}{K_S T_m'}, T_{ip} = 2 \cdot T_m'$ $T_{dp} = 0,5 \cdot T_m'$
2	izmerjeni vrednosti K_{pkr} in T_{kr}	P	$K_p = 0,5 \cdot K_{pkr}$
		PI	$K_p = 0,45 \cdot K_{pkr}, T_{ip} = 0,83 \cdot T_{kr}$
		PID	$K_p = 0,6 \cdot K_{pkr}, T_{ip} = 0,5 \cdot T_{kr}$ $T_{dp} = 0,125 \cdot T_{kr}$

Simetrični optimum

Priladen za regulacije s konstantno želeno vrednostjo (dober odziv na motnje)

	Regulirane	Reg.	Izračun parametrov regulatorja
1	$F_S = \frac{K_S}{(1+sT_1)(1+sT_\mu)}$ ali $F_S = \frac{K_S}{sT_1(1+sT_\mu)}$ $T_1 \gg T_\mu$	PI	$K_p = \frac{T_1}{2K_S T_\mu}$
	$T_1 > T_2 \gg T_\mu$		$T_{ip} = 4T_\mu$
2	$F_S = \frac{K_S}{(1+sT_1)(1+sT_2)(1+sT_\mu)}$ ali $F_S = \frac{K_S}{sT_1(1+sT_2)(1+sT_\mu)}$ $T_1 > T_2 \gg T_\mu$	PID	$K_p = \frac{T_1}{2K_S T_\mu}$
			$T_{ip} = 4T_\mu$ $T_{dp} = T_2$

	Nonlinearni člen	Enačba kritične trajektorije
1	člen z nasičenjem $k = \frac{x_o}{y_o}$	$R(y_1) = -\frac{\pi y_1}{2x_o} \cdot \frac{1}{\arcsin \frac{y_o}{y_1} + \frac{y_o}{y_1} \sqrt{1 - \left(\frac{y_o}{y_1}\right)^2}}$
2	dvopolozajni člen z mrtvo cono	$R(y_1) = -\frac{\pi y_1}{4x_o} \cdot \frac{1}{\sqrt{1 - \left(\frac{\Delta}{2y_1}\right)^2}}$
3	dvopolozajni člen	$R(y_1) = -\frac{\pi y_1}{4x_o}$
4	člen s histerezo	$R(y_1) = -\frac{1}{\beta(y_1)} \cdot e^{-j(\psi(y_1)+\pi)}$ $\beta(y_1) = \frac{\sqrt{A_1^2 + B_1^2}}{y_1}, \psi(y_1) = \arctan \frac{A_1}{B_1}$ $A_1 = \frac{4ky_1}{\pi} \cdot \left[\frac{(h/2)^2}{y_1^2} - \frac{(h/2)}{y_1} \right]$ $B_1 = \frac{ky_1}{\pi} \cdot \left[\frac{\pi}{2} + \arcsin \frac{y_1 - h}{y_1} + \frac{h \cdot (y_1 - h)}{y_1^2} \sqrt{\frac{2y_1}{h} - 1} \right]$

Statični pogrešek

$$\varepsilon = x^* - x$$

$$\varepsilon_s = \varepsilon(t = \infty)$$

$$\varepsilon_s = x^* (t = \infty) - x(t = \infty)$$

$$\varepsilon_s = \lim_{s \rightarrow 0} [s \cdot z(s) \cdot H_M] = \lim_{s \rightarrow 0} [s \cdot z(s) \cdot \frac{F_M}{1 + F_o}]$$

$$\varepsilon_s = \lim_{s \rightarrow 0} [s \cdot x^*(s)] - \lim_{s \rightarrow 0} [s \cdot x(s)]$$

$$H = \frac{x(s)}{x^*(s)} \quad (z = 0)$$

$$x(s) = x^*(s) \cdot H$$

$$\varepsilon_s = \lim_{s \rightarrow 0} [s \cdot x^*(s)] - \lim_{s \rightarrow 0} [s \cdot x^*(s) \cdot H]$$

$$\varepsilon_s = \lim_{s \rightarrow 0} [s \cdot x^*(s) \cdot (1 - H)]$$

Za motnjo:

$$\varepsilon_s = \lim_{s \rightarrow 0} [s \cdot z(s) \cdot H_M] = \lim_{s \rightarrow 0} [s \cdot z(s) \cdot \frac{F_M}{1 + F_o}]$$

Za direktno povratno zanko $H = \frac{F_o}{1 + F_o}$:

$$\varepsilon_s = \lim_{s \rightarrow 0} [s \cdot x^*(s) \cdot \left(\frac{1}{1 + F_o} \right)]$$

Za sistem brez integratorjev v direktni veji in $x^* = \frac{1}{s}$:

$$\varepsilon_s = \frac{1}{1 + K_o}$$

Routhov kriterij:

Karakteristična enačba: $a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + a_{n-2} \cdot s^{n-2} + \dots + a_1 \cdot s^1 + a_0 \cdot s^0 = 0$

s^n	a_n	a_{n-2}	a_{n-4}	...
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	...
s^{n-2}	$\frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$	$\frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$		

Hurwitzov kriterij:

Karakteristična enačba (polinom) $a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + a_{n-2} \cdot s^{n-2} + \dots + a_1 \cdot s^1 + a_0 \cdot s^0 = 0$:
- vsi koeficienti istega predznaka in različni od 0
- vse determinante Δ_k morajo biti večje od 0 ($n-2$ determinant!)

$$\Delta_k = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \dots & a_{n+1-2k} \\ a_n & a_{n-2} & a_{n-4} & \dots & a_{n+2-2k} \\ 0 & a_{n-1} & a_{n-3} & \dots & a_{n+3-2k} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_{n-k} \end{vmatrix} > 0$$

$$\lambda = [2 \dots n-1]$$

Za karakteristično enačbo 3. reda $a_3 \cdot s^3 + a_2 \cdot s^2 + a_1 \cdot s^1 + a_0 \cdot s^0 = 0$:

$$\Delta_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} > 0$$

Optimum iznos

Priladen za vodene regulacije (dober odziv na spremembe želene vrednosti)

	Regulirane	Reg.	Izračun parametrov regulatorja
1	$F_S = \frac{K_S}{1+sT_1}$	I	$T_i = 2K_S T_1$
2	$F_S = \frac{K_S}{(1+sT_1)(1+sT_\mu)}$ $T_1 \gg T_\mu$	I	$T_i = 2K_S T_1$
		PI	$K_p = \frac{T_1}{2K_S T_\mu}$ $T_{ip} = T_1$
3	$F_S = \frac{K_S}{(1+sT_1)(1+sT_2)(1+sT_\mu)}$ $T_1 > T_2 \gg T_\mu$	I	$T_i = 2K_S (T_1 + T_2)$
		PI	$K_p = \frac{T_1^2 + T_2^2}{2K_S T_1 T_2}$ $T_{ip} = \frac{(T_1^2 + T_2^2)(T_1 + T_2)}{T_1^2 + T_1 T_2 + T_2^2}$
		PID	$K_p = \frac{T_1}{2K_S T_\mu}$ $T_{ip} = T_1$ $T_{dp} = T_2$
4	$F_S = \frac{e^{-\tau_i}}{1+sT_2}$ $A = \frac{T_1}{T_2}$	P	$K_p = \frac{A^2}{1+2A}$
		I	$T_i = 2(T_1 + T_2)$
		PI	$K_p = \frac{1}{4} \frac{6A^3 + 6A^2 + 3A + 1}{3A^2 + 3A + 1}$ $T_{ip} = \frac{T_1}{3} \frac{6A^3 + 6A^2 + 3A + 1}{2A^2 + 2A + 1}$
			za $A > 2$: $K_p \approx \frac{T_2}{2T_1} + \frac{T_1}{12T_2}, T_{ip} \approx T_2 + \frac{T_1^3}{6T_2^2}$
PID	$K_p = \frac{1}{16} \frac{180A^4 + 240A^3 + 135A^2 + 42A + 7}{15A^4 + 15A^3 + 6A + 1}$ $T_{ip} = \frac{T_1}{15} \frac{180A^4 + 240A^3 + 135A^2 + 42A + 7}{12A^3 + 12A^2 + 5A + 1}$ $T_{dp} = \frac{T_2}{16} \frac{60A^4 + 60A^3 + 27A^2 + 7A + 1}{15A^4 + 15A^3 + 6A + 1}$		
	za $A > 2$: $K_p \approx \frac{3T_1}{4T_2} + \frac{1}{4} \frac{T_1}{80T_2^2}, T_{ip} = T_2 + \frac{T_1}{3}, T_{dp} \approx \frac{T_2}{4} + \frac{T_1^2}{80T_2}$		