Laplaceova transformacija

$$\mathcal{L}\{y(t)\}=Y(s) \qquad Y(s)=\int_{0^-}^{\infty} y(t)e^{-st}dt \qquad Y(s)=F(s)U(s) \qquad y(t)=\frac{1}{2\pi j}\int_{\sigma_c-j\infty}^{\sigma_c+j\infty} Y(s)e^{st}ds$$

Lastnosti Laplaceove transformacije

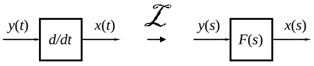
Superpozicija:	$\mathcal{L}\{a_1y_1(t)+a_2y_2(t)\}=a_1Y_1(s)+a_2Y_2(s)$
Homogenost:	$\mathcal{L}\{ay(t)\}=aY(s)$
Časovni premik:	$\mathcal{L}\{y(t-T)\}=e^{-sT}Y(s)$
Časovno skaliranje:	$\mathcal{L}\{y(at)\}=\frac{1}{\lvert a\rvert}Y\left(\frac{s}{a}\right)$
Frekvenčni premik:	$\mathcal{L}\{e^{-at}y(t)\}=Y(s+a)$
Odvajanje:	$\mathcal{L}\left\{\frac{dy}{dt}\right\}=-y(t=0^-)+sY(s)$ $\mathcal{L}\left\{\frac{d^ny}{dt^n}\right\}=s^nY(s)-s^{n-1}y(t=0^-)-s^{n-2}\frac{d^1y}{dt^1}(t=0^-)-s^{n-3}\frac{d^2y}{dt^2}(t=0^-)-...-\frac{d^{n-1}y}{dt^{n-1}}(t=0^-)$
Integriranje:	$\mathcal{L}\left\{\int_0^t y(\tau)d\tau\right\}=\frac{1}{s}Y(s)$
Konvolucija v časovnem prostoru ustreza množenju v frekvenčnem prostoru:	$\mathcal{L}\{y_1(t)*y_2(t)\}=Y_1(s)Y_2(s)$
Produkt v čas. prostoru ustreza konvoluciji v frekv. prostoru:	$\mathcal{L}\{y_1(t)y_2(t)\}=\frac{1}{2\pi j}\int_{\sigma_c-j\infty}^{\sigma_c+j\infty}Y_1(\xi)Y_2(s-\xi)d\xi$
Množenje s časom ustreza odvajanju v frekvenčnem prostoru:	$\mathcal{L}\{ty(t)\}=-\frac{d}{ds}Y(s)$

Tabela Laplaceovih transformov

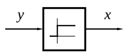
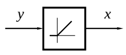
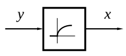
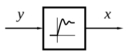
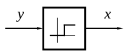
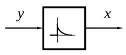
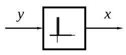
Oznake: $A=\sqrt{a^2+b^2}$ $\quad \omega=\sqrt{b^2-a^2}$

$Y(s)$	$y(t)$
1	$\delta(t)$
$\frac{1}{s}$	1
$\frac{1}{s^2}$	t
$\frac{1}{s^n};n>0$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{s-a}$	e^{at}
$\frac{1}{1+sa}$	$\frac{1}{a}e^{-t/a}$
$\frac{1}{s(s-a)}$	$\frac{1}{a}(e^{at}-1)$
$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$
$\frac{1}{s(1+as)}$	$1-e^{-t/a}$
$\frac{1}{(s-a)^n};n>0$	$\frac{t^{n-1}}{(n-1)!}e^{at}$
$\frac{1}{(s-a)(s-b)}$	$\frac{e^{at}-e^{bt}}{a-b}$
$\frac{1}{(1+as)(1+bs)}$	$\frac{e^{-t/a}-e^{-t/b}}{a-b}$
$\frac{1}{s^2+a^2}$	$\frac{1}{a}\sin(at)$
$\frac{1}{s^2-a^2}$	$\frac{1}{a}\sinh(at)$
$\frac{s}{s^2+a^2}$	$\cos(at)$
$\frac{s}{s^2-a^2}$	$\cosh(at)$
$\frac{1}{s^2+2as+b^2}$	$\frac{1}{\omega}e^{-at}\sin(\omega t)$
$\frac{s}{s^2+2as+b^2}$	$e^{-at}\left[\cos(\omega t)-\frac{a}{\omega}\sin(\omega t)\right]$
$\frac{1}{s(s^2+2as+b^2)}$	$\frac{1}{b^2}\left\{1-e^{-at}\left[\cos(\omega t)+\frac{a}{\omega}\sin(\omega t)\right]\right\}$
$\frac{1}{(as+1)^3}$	$\frac{1}{2a^3}t^2e^{-t/a}$
$\frac{1}{s(s^2+a^2)}$	$\frac{1}{a^2}[1-\cos(at)]$
$\frac{as}{(s^2+a^2)^2}$	$\frac{t}{2}\sinh(at)$

Osnovni linearni členi v regulacijskih sistemih



$$F(s)=\frac{x(s)}{y(s)}=K\frac{\prod_{l=1}^{D}(1+sT_l)\cdot\prod_{m=1}^{E}(1+2z_mT_ms+T_m^2s^2)}{s^{\pm A}\cdot\prod_{n=1}^{B}(1+sT_n)\cdot\prod_{r=1}^{C}(1+2z_rT_rs+T_r^2s^2)}$$

Ime člena	Blok	Prenosna funkcija $F(s)=\frac{x(s)}{y(s)}$	Zapis v časovnem prostoru (diferencialna enačba)	Opomba	Zgled
Proporcionalni člen		$F(s)=K$	$x(t)=K\cdot y(t)$	K ... ojačenje	uporovni delilnik; idealni ojačevalnik
Integralni člen		$F(s)=\frac{1}{sT_i}$	$x(t)=\frac{1}{T_i}\int_0^t y(\tau)\cdot d\tau$	T_i ... integracijska časovna konstanta	zasuk je integral vrtilne hitrosti (motor)
Člen 1. reda		$F(s)=\frac{K}{1+sT}$	$\frac{dx}{dt}T+x(t)=K\cdot y(t)$	K ... ojačenje T ... časovna konstanta	polnjenje kondenzatorja
Člen 2. reda		$F(s)=\frac{K}{1+2zTs+s^2T^2}$	$\frac{d^2x}{dt^2}T^2+\frac{dx}{dt}2zT+x=K\cdot y$	K ... ojačenje T ... časovna konstanta z ... faktor dušenja	RLC nihajni krog
Člen z mrtvim časom		$F(s)=K\cdot e^{-sT_m}$	$x(t)=K\cdot y(t-T_m)$	K ... ojačenje T_m ... mrtvi čas	tekoči trak
Realni diferencialni člen		$F(s)=\frac{sT_d}{1+sT_d'}$	$x(t)+\frac{dx}{dt}T_d'=\frac{dy}{dt}T_d$	T_d ... časovna konstanta T_d' ... parazitna časovna konstanta	praznjenje kondenzatorja
Idealni diferencialni člen		$F(s)=sT_d$	$x(t)=\frac{dy}{dt}T_d$	T_d ... časovna konstanta $(T_d'=0)$	prehodna funkcija: Diracov impulz

Člen 1. reda:

$K=\alpha(\omega=0)$ $\qquad T=\frac{1}{\omega_o}$ $\qquad \omega_o=\omega(\varphi=-45°)$

Člen 2. reda:

$K=\alpha(\omega=0)$ $\qquad T=\frac{1}{\omega_o}$ $\qquad \omega_o=\omega(\varphi=-90°)$ $\qquad \omega_r=\omega_o\sqrt{1-2z^2}$ $\qquad \omega_l=\omega_o\sqrt{1-z^2}$

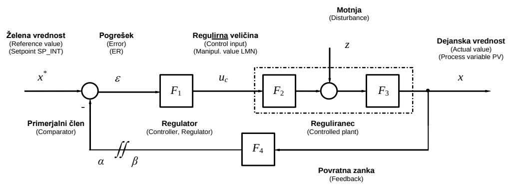
$\alpha_{\max}=K\cdot Q_r$ $\qquad Q_r=\frac{1}{2z\sqrt{1-z^2}}$ $\qquad z=\sqrt{\frac{1\mp\sqrt{1-1/Q_r^2}}{2}}$ $\qquad T=\frac{1}{\omega_o}=\frac{T_l\sqrt{1-z^2}}{2\pi}$

$\alpha(\omega_o)=\alpha(\varphi=-90°)=\frac{K}{2z}$ $\qquad x_{\max}=K(1+e^{\frac{-\pi z}{\sqrt{1-z^2}}})$ $\qquad t(x=x_{\max})=\frac{\pi}{\omega_l}$

Frekvenčna karakteristika:

$$\alpha(\omega)=\lvert F(j\omega)\rvert=\sqrt{(\mathrm{Re}[F(j\omega)])^2+(\mathrm{Im}[F(j\omega)])^2} \qquad \varphi(\omega)=\mathrm{arctg}\left(\frac{\mathrm{Im}(F(j\omega))}{\mathrm{Re}(F(j\omega))}\right)$$

Prenosne funkcije odprtega in zaključenega regulacijskega kroga



$F_o = -\frac{\beta}{\alpha}\Big|$(na mestu prekinitve, $x^* = 0,\ z = 0$) $\qquad F_o = F_1 \cdot F_2 \cdot F_3 \cdot F_4$

$H = \frac{x}{x^*}\Big|$($z = 0$, sklenjena povratna zanka) $\qquad H = \frac{F_1 \cdot F_2 \cdot F_3}{1 + F_1 \cdot F_2 \cdot F_3 \cdot F_4} = \frac{F_1 \cdot F_2 \cdot F_3}{1 + F_o}$

$F_M = \frac{x}{z}\Big|$($x^* = 0$, prekinjena povratna zanka) $\qquad F_M = F_3$

$H_M = \frac{x}{z}\Big|$($x^* = 0$, sklenjena povratna zanka) $\qquad H_M = \frac{F_3}{1 + F_1 \cdot F_2 \cdot F_3 \cdot F_4} = \frac{F_M}{1 + F_o}$

Za direktno povratno zanko (če je $F_4 = 1$) velja tudi:

$F_o = \frac{x}{x^*}\Big|$($z = 0$, prekinjena povratna zanka) $\qquad F_o = F_1 \cdot F_2 \cdot F_3$

$H = \frac{x}{x^*}\Big|$($z = 0$, sklenjena povratna zanka) $\qquad H = \frac{F_1 \cdot F_2 \cdot F_3}{1 + F_1 \cdot F_2 \cdot F_3} = \frac{F_o}{1 + F_o}$

$F_M = \frac{x}{z}\Big|$($x^* = 0$, prekinjena povratna zanka) $\qquad F_M = F_3$

$H_M = \frac{x}{z}\Big|$($x^* = 0$, sklenjena povratna zanka) $\qquad H_M = \frac{F_3}{1 + F_1 \cdot F_2 \cdot F_3} = \frac{F_M}{1 + F_o}$

Optimiziranje parametrov regulatorjev

P-regulator: $F_R = K_p$ $\qquad$ I-regulator: $F_R = \frac{1}{sT_i}$

PI-regulator: $F_R = K_p \frac{1 + sT_{ip}}{sT_{ip}}$ $\qquad$ PID-regulator: $F_R = K_p \frac{(1 + sT_{ip})(1 + sT_{dp})}{sT_{ip}}$

Priporočila po Ziegler-Nicholsu za procesne regulacije

	Reguliranec	Reg.	Izračun parametrov regulatorja
1	$F_S \approx e^{-sT_m'} \frac{K_S}{(1 + sT')}$	P	$K_p = \frac{T'}{K_S T_m'}$
		PI	$K_p = 0{,}9 \cdot \frac{T'}{K_S T_m'}$ $\quad T_{ip} = 3{,}3 \cdot T_m'$
		PID	$K_p = 1{,}2 \cdot \frac{T'}{K_S T_m'}$ $\quad T_{ip} = 2 \cdot T_m'$ $T_{dp} = 0{,}5 \cdot T_m'$
2	izmerjeni vrednosti K_{pkr} in T_{kr}	P	$K_p = 0{,}5 \cdot K_{pkr}$
		PI	$K_p = 0{,}45 \cdot K_{pkr}$ $\quad T_{ip} = 0{,}83 \cdot T_{kr}$
		PID	$K_p = 0{,}6 \cdot K_{pkr}$ $\quad T_{ip} = 0{,}5 \cdot T_{kr}$ $T_{dp} = 0{,}125 \cdot T_{kr}$

Simetrični optimum

Prikladen za regulacije s konstantno želeno vrednostjo (dober odziv na motnje)

	Reguliranec	Reg.	Izračun parametrov regulatorja
1	$F_S = \frac{K_S}{(1 + sT_1)(1 + sT_\mu)}$ ali $F_S = \frac{K_S}{sT_1(1 + sT_\mu)}$ $T_1 >> T_\mu$	PI	$K_p = \frac{T_1}{2K_S T_\mu}$ $T_{ip} = 4T_\mu$
2	$F_S = \frac{K_S}{(1 + sT_1)(1 + sT_2)(1 + sT_\mu)}$ ali $F_S = \frac{K_S}{sT_1(1 + sT_2)(1 + sT_\mu)}$ $T_1 > T_2 >> T_\mu$	PID	$K_p = \frac{T_1}{2K_S T_\mu}$ $T_{ip} = 4T_\mu$ $T_{dp} = T_2$

	Nelinearni člen	Enačba kritične trajektorije
1	člen z nasičenjem $k = \frac{x_o}{y_o}$	$R(y_1) = -\frac{\pi y_o}{2x_o} \cdot \frac{1}{\arcsin\frac{y_o}{y_1} + \frac{y_o}{y_1}\sqrt{1 - \left(\frac{y_o}{y_1}\right)^2}}$
2	dvopoložajni člen z mrtvo cono	$R(y_1) = -\frac{\pi y_1}{4x_o} \cdot \frac{1}{\sqrt{1 - \left(\frac{\Delta}{2y_1}\right)^2}}$
3	dvopoložajni člen	$R(y_1) = -\frac{\pi y_1}{4x_o}$
4	člen s histerezo	$R(y_1) = -\frac{1}{\beta(y_1)} \cdot e^{-j(\psi(y_1) + \pi)}$ $\beta(y_1) = \frac{\sqrt{A_1^2 + B_1^2}}{y_1}$, $\psi(y_1) = \arctan\frac{A_1}{B_1}$ $A_1 = \frac{4ky_1}{\pi} \cdot \left[\frac{(h/2)^2}{y_1^2} - \frac{(h/2)}{y_1}\right]$ $B_1 = \frac{ky_1}{\pi} \cdot \left[\frac{\pi}{2} + \arcsin\frac{y_1 - h}{y_1} + \frac{h \cdot (y_1 - h)}{y_1^2}\sqrt{\frac{2y_1}{h} - 1}\right]$

Statični pogrešek

$\varepsilon = x^* - x$

$\varepsilon_s = \varepsilon(t = \infty)$

$\varepsilon_s = x^*(t = \infty) - x(t = \infty)$

$\varepsilon_s = \lim_{s \to 0}[s \cdot x^*(s)] - \lim_{s \to 0}[s \cdot x(s)]$

$H = \frac{x(s)}{x^*(s)}\Big|(z = 0)$

$x(s) = x^*(s) \cdot H$

$\varepsilon_s = \lim_{s \to 0}[s \cdot x^*(s)] - \lim_{s \to 0}[s \cdot x^*(s) \cdot H]$

$\varepsilon_s = \lim_{s \to 0}[s \cdot x^*(s) \cdot (1 - H)]$

Za motnjo:

$\varepsilon_s = \lim_{s \to 0}[s \cdot z(s) \cdot H_M] = \lim_{s \to 0}\left[s \cdot z(s) \cdot \frac{F_M}{1 + F_o}\right]$

Za direktno povratno zanko $H = \frac{F_o}{1 + F_o}$:

$\varepsilon_s = \lim_{s \to 0}\left[s \cdot x^*(s) \cdot \left(\frac{1}{1 + F_o}\right)\right]$

Za sistem brez integratorjev v direktni veji in $x^* = \frac{1}{s}$:

$\varepsilon_s = \frac{1}{1 + K_o}$

Routhov kriterij:

Karakteristična enačba: $a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + a_{n-2} \cdot s^{n-2} ... + a_1 \cdot s^1 + a_0 \cdot s^0 = 0$

s^n	:	a_n	a_{n-2}	a_{n-4}	...
s^{n-1}	:	a_{n-1}	a_{n-3}	a_{n-5}	...
s^{n-2}	:	$\frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$	$\frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$		

Hurwitzov kriterij:

Karakteristična enačba (polinom) $a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + a_{n-2} \cdot s^{n-2} ... + a_1 \cdot s^1 + a_0 \cdot s^0 = 0$:

- vsi koeficienti istega predznaka in *različni* od 0
- vse determinante Δ_λ morajo biti večje od 0 (n-2 determinant!)

$$\Delta_\lambda = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} & ... & a_{n+1-2\lambda} \\ a_n & a_{n-2} & a_{n-4} & ... & a_{n+2-2\lambda} \\ 0 & a_{n-1} & a_{n-3} & ... & a_{n+3-2\lambda} \\ 0 & a_n & a_{n-2} & ... & a_{n+4-2\lambda} \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & a_{n-\lambda} \end{vmatrix} > 0$$

$$\lambda = [2\ ...\ n-1]$$

Za karakteristično enačbo 3. reda $a_3 \cdot s^3 + a_2 \cdot s^2 + a_1 \cdot s^1 + a_0 \cdot s^0 = 0$:

$$\Delta_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} > 0$$

Optimum iznosa

Prikladen za vodene regulacije (dober odziv na spremembe želene vrednosti)

	Reguliranec	Reg.	Izračun parametrov regulatorja
1	$F_S = \frac{K_S}{1 + sT_1}$	I	$T_i = 2K_S T_1$
2	$F_S = \frac{K_S}{(1 + sT_1)(1 + sT_\mu)}$ $T_1 >> T_\mu$	I	$T_i = 2K_S T_1$
		PI	$K_p = \frac{T_1}{2K_S T_\mu}$ $T_{ip} = T_1$
3	$F_S = \frac{K_S}{(1 + sT_1)(1 + sT_2)(1 + sT_\mu)}$ $T_1 > T_2 >> T_\mu$	I	$T_i = 2K_S(T_1 + T_2)$
		PI	$K_p = \frac{T_1^2 + T_2^2}{2K_S T_1 T_2}$ $T_{ip} = \frac{(T_1^2 + T_2^2)(T_1 + T_2)}{T_1^2 + T_1 T_2 + T_2^2}$
		PID	$K_p = \frac{T_1}{2K_S T_\mu}$ $T_{ip} = T_1$ $T_{dp} = T_2$
4	$F_S = \frac{e^{-sT_1}}{1 + sT_2}$ $A = \frac{T_1}{T_2}$	P	$K_p = \frac{A^2}{1 + 2A}$
		I	$T_i = 2(T_1 + T_2)$
		PI	$K_p = \frac{1}{4} \cdot \frac{6A^3 + 6A^2 + 3A + 1}{3A^2 + 3A + 1}$ $T_{ip} = \frac{T_1}{3} \cdot \frac{6A^3 + 6A^2 + 3A + 1}{2A^2 + 2A + 1}$ za $A > 2$: $K_p \approx \frac{T_2}{2T_1} + \frac{T_1}{12T_2}$, $T_{ip} \approx T_2 + \frac{T_1^3}{6T_2^2}$
		PID	$K_p = \frac{1}{16} \cdot \frac{180A^4 + 240A^3 + 135A^2 + 42A + 7}{15A^3 + 15A^2 + 6A + 1}$ $T_{ip} = \frac{T_1}{15} \cdot \frac{180A^4 + 240A^3 + 135A^2 + 42A + 7}{12A^3 + 12A^2 + 5A + 1}$ $T_{dp} = \frac{T_2}{16} \cdot \frac{60A^4 + 60A^3 + 27A^2 + 7A + 1}{15A^3 + 15A^2 + 6A + 1}$ za $A > 2$: $K_p \approx \frac{3}{4}\frac{T_1}{T_2} + \frac{1}{4} + \frac{T_1}{80T_2}$, $T_{ip} \approx T_2 + \frac{T_1}{3}$ $\ T_{dp} \approx \frac{T_2}{4} + \frac{T_1^2}{80T_2}$